Channel-Dependent Scheduling of Uplink Single Carrier FDMA Systems

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Abstract – **We examine single carrier frequency division multiple access (SC-FDMA) with frequency domain equalization for uplink data transmission. We investigate channel-dependent scheduling schemes to achieve multi-user diversity and frequency selective diversity. There are two subcarrier mapping schemes in SC-FDMA: *Localized FDMA* (L-FDMA) and *Interleaved FDMA* (I-FDMA). L-FDMA benefits from frequency selective scheduling, but it incurs higher peak-to-average power ratio than I-FDMA. Throughout our work, we provide low complexity channel-dependent scheduling (CDS) methods for L-FDMA and I-FDMA. The results show that rate-sum capacity can increase up to 130% for L-FDMA and 40% for I-FDMA relative to static round robin scheduling.**

I. Introduction

Orthogonal frequency division multiple access (OFDMA) is a promising multiple access scheme in the future generation wireless communication systems such as Long Term Evolution of 3GPP and IEEE 802.16e [1,2]. Despite many benefits of OFDMA for high speed data services, they suffer from high envelope fluctuation in the time domain, leading to large peak–to-average-power ratio (PAPR). Because high PAPR is detrimental to mobile terminals, SC-FDMA has drawn great attention as an attractive alternative to OFDMA for uplink data transmission [3]. It can be viewed as DFT-spread OFDMA, where time domain data symbols are transformed to frequency domain by a discrete Fourier transform (DFT) before going through OFDMA modulation.

In this paper, we focus on channel-dependent scheduling (CDS) of uplink SC-FDMA systems. In radio environment, channels with wide bandwidth may experience frequency selective fading. When a user sends information to the base station, the channel gains are different for different subcarriers and the channels are uncorrelated or less correlated for different users. Thereby, some subcarriers which are in deep frequency selective fading for one user may be in an excellent condition for other users. The scheduler in the base station can assign the subcarriers to a favorable user so that the frequency selective fading can provide beneficial diversity to the system.

SC-FDMA has two types of subcarrier mapping: Localized FDMA (L-FDMA) and Interleaved FDMA (I-FDMA). In L-FDMA, the scheduler assigns consecutive subcarriers to convey information from a particular user. Thus L-FDMA achieves frequency selective diversity if it assigns each user to subcarriers in a portion of the signal band where that user has favorable transmission characteristics. In I-FDMA, users are assigned subcarriers that are distributed over the entire frequency band in order to avoid allocating many adjacent subcarriers in deep fading. By selecting users which are in favorable channel condition over the entire bandwidth, we obtain multi-user diversity in I-FDMA system.

Concerning channel-dependent scheduling, most of the previous work has focused on power and subcarrier allocation in downlink OFDMA systems [4,5,6]. C. Y. Wong et al. proposed optimal bit, power, and subcarrier allocations for real-time users requiring a constant data rate. However, the proposed method is inadequate for practical implementations, since lots of iterations are required with high complexity. J. Jang and K. B. Lee found that the data rate of OFDMA is maximized when each subcarrier is assigned to only one user with the best channel gain. They proposed a simpler method of subcarrier and power allocation with two steps. One is to assign the subcarriers to the users with the best channel gains. Then, the transmit power of each user is distributed over the assigned subcarriers by a water-filling policy. Seeking optimal allocation algorithms in uplink transmission by solving a standard form of optimization problem becomes more complex, since an additional power constraint for each user is added into the optimization problem. K. Kim et al.[11] seeks joint subcarrier and power allocation in uplink OFDMA. Instead of finding optimal solutions by solving the combined problem of power and subcarrier allocations, they proposed a simpler sub-optimal algorithm based on "greedy" allocation.

The previous work related to SC-FDMA has been mainly focused on implementation problems in the physical layer [3,7,8]. Our previous work [3] compares PAPR characteristics using complementary cumulative distribution function (CCDF) of PAPR of OFDMA, I-FDMA, and L-FDMA. We found that SC-FDMA signals indeed have lower PAPR compared to those of OFDMA. Also, we showed that L-FDMA incurs higher PAPR compared to I-FDMA. M. Schnell [7] proposed maximum likelihood detection for I-FDMA system and showed the improvement in BER performance. R. Dinis [8] examines multiple access methods of I-FDMA that allow users with different data rates to transmit simultaneously. References [9,10] derive an effective

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SNR and capacity of SC-FDMA with frequency domain equalizers.

Our objective in this paper is to derive low complexity algorithms for CDS to provide higher data rate in uplink communications. The algorithms consist of subcarrier or chunk (a subset of subcarriers) allocations and power allocations for multiple chunks with constrained transmit power in mobile terminals. Since SC-FDMA uses single carrier modulation, it suffers from inter-symbol interference (ISI). To combat the ISI, we consider a SC-FDMA system with a minimum mean square error (MMSE) frequency domain equalization.

This paper is organized as follows: Section II presents a basic description of the SC-FDMA system and measures of system capacity when a MMSE frequency domain equalizer is employed in the receiver. Section III describes CDS methods for both I-FDMA and L-FDMA. The system analysis including the scheduling schemes and performance measures is discussed in section IV. Section V presents conclusions.

II. SC-FDMA SYSTEMS

A. Subcarrier mapping

With respect to resource allocation, we consider that a subset of subcarriers comprises a chunk, and one or more chunks are allocated to each user, since coordinating individual subcarriers is too burdensome. Thus, the number of subcarriers in a chunk is regarded as a minimum resource unit for subcarrier allocation. As mentioned earlier, there are two types of subcarrier mapping: L-FDMA and I-FDMA. A chunk in L-FDMA consists of consecutive subcarriers. Subcarriers in a chunk of I-FDMA are distributed over the entire bandwidth with equal frequency spacing.

We consider time synchronized uplink transmission with the system bandwidth B Hz, multiplexed in time and frequency (subcarrier) division. The total subcarriers (L) are divided into several chunks so that each chunk is allocated to a user by the scheduler in the base station. The number of subcarriers per chunk is $M= L/N$, where N denotes the number of chunks. Each user can transmit M data symbols per chunk per time interval. Fig. 1 shows an example of chunk structure for I-FDMA and L-FDMA, where there are 16 subcarriers and 4 chunks.

The base station estimates channel gains for all users and selects the users allowed to transmit simultaneously in each time interval, based on the quality of previously received signals or reference signals transmitted in advance. We assume that the base station has perfect knowledge of channel gains of all users in the frequency domain. The data constellations of the allocated users are also determined in the base station, and transmitted to the terminals via downlink control signals.

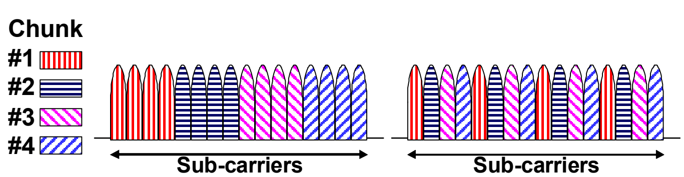

Fig. 1 An example of chunk structures (L:16,M:4) (a) L-FDMA (b) I-FDMA

B. Chunk capacity of SC-FDMA

SC-FDMA uses a frequency domain equalizer to mitigate ISI. We assume that a MMSE equalizer is used, and from [9,10] the SNR of data delivered in a chunk with MMSE equalization can be written as (1), if an arbitrary subset of subcarrier, I (a chunk contains M subcarriers) is assigned to user k. $\gamma_{i,k}$ is the SNR of subcarrier i for user k.

$$\gamma_k = \left(\frac{1}{\frac{1}{M}\sum_{i\in I}\frac{\gamma_{i,k}}{\gamma_{i,k}+1}} - 1 \right)^{-1} \tag{1}$$

Using Shannon's formula, the achievable data rate of the chunk for user k has the upper bound

$$C_k = (B/N)\log_2(1+\gamma_k) \tag{2}$$

Note that the effective bandwidth occupied by user k is B/N Hz, since one chunk is allocated to user k and there are N chunks in bandwidth B Hz.

C. Bit and power allocation for multiple chunks

In general, different transmit powers or different constellations may be allocated to the different chunks when a user occupies multiple chunks. Similar to OFDMA, chunk-based greedy power and bit loading may be employed with SC-FDMA. However, the improvement in throughput may not be significant enough to justify the added complexity. As a realistic solution, we consider equal-bit-equal-power (EBEP) allocation for each chunk. Thus, we assume that the power assigned to each subcarrier is determined as $P_k^{(sub)}=P_k/|I_{sub,k}|$, where P_k is the total transmit power of user k, $I_{sub,k}$ is the subcarrier index set assigned to user k, and $|I_{sub,k}|$ is the number of subcarriers assigned to user k. Note that the assigned subcarrier set of user k, $I_{sub,k}$, is the union of the subset of subcarriers in each of the chunks assigned to user k. $I_{ch,k}$ is the assigned chunk set of user k and $I_{sub}^{(n)}$ denotes the set of subcarriers in chunk n.

$$I_{sub,k} = \bigcup_{n\in I_{ch,k}} I_{sub}^{(n)} \tag{3}$$

Then, the SNR for the data delivered with chunks $I_{ch,k}$ can be derived as (4). σ_i^2 is the noise power of subcarrier i, and $H_{i,k}$ is the channel gain of subcarrier i for user k.

$$\gamma(P_k, I_{ch,k}) = \left(\frac{1}{\frac{1}{|I_{sub,k}|}\sum_{i\in I_{sub,k}}\frac{\gamma_{i,k}}{\gamma_{i,k}+1}} - 1 \right)^{-1}, \gamma_{i,k} = \frac{P_k^{(sub)} H_{i,k}}{\sigma_i^2} \tag{4,5}$$

Similar to (2), the upper bound on data rate of user k is

$$C_k(P_k, I_{ch,k}) = \frac{B|I_{ch,k}|}{N}\cdot\log_2\left[1+\gamma(P_k, I_{ch,k})\right] \tag{6}$$

III. CHANNEL DEPENDENT SCHEDULING WITH EQUAL BIT EQUAL POWER PER CHUNK

A. Localized FDMA

The upper bound on data rate of user k with allocated chunks $I_{ch,k}$ is represented as $C_k(P_k, I_{ch,k})$. Then, a general optimization problem for multiple users (K users) can be represented as

$$\max \sum_{k=1}^{K} C_k\left(P_k, I_{ch,k}\right) \tag{7}$$

The orthogonality of the users stems from the fact that each user occupies different subcarriers as specified in (8).

$$\text{If } n \in I_{ch,j}\text{, then } n \notin I_{ch,k} \text{ for } j,k \in \{1,2,\cdots,\mathrm{K}\}, j \neq k \tag{8}$$

For EBEP allocation, the transmit power of each subcarrier $P_k^{(sub)}$ is determined by (9).

$$P_k = P_{\max} \rightarrow P_k^{(sub)} = \frac{P_{\max}}{\left|I_{sub,k}\right|} \tag{9}$$

Finding the optimum solution requires combinatorial comparisons for all possible chunk allocations. In this paper, we provide a tractable greedy chunk allocation similar to the method derived in [11] using the following procedure.

- Sub-optimal chunk allocation : Greedy allocation

Initialization: All users and chunks are added into the sets of available users and chunks i.e. $I_{avail\_user}=\{1,2,\ldots,K\}$, $I_{avail\_chunk}=\{1,2,\ldots,N\}$.

Step 1(Chunk selection): Find a chunk which has the highest channel gain among all available chunks and users. For each available user j and chunk n, find the pair

$$\left[n^*, j^*\right] = \arg \max_{\substack{n \in I_{avail\_chunk} \\ j \in I_{avail\_user}}} C_j\left(P_{\max}, \{n\}\right) \tag{10}$$

where {n} is the set containing the single chunk n.

Step 2 (Greedy chunk allocation): Find a user (k) who can maximize the marginal capacity with the chunk selected in step 1. Then, allocate chunk n* to user k* as follows.*

$$\tilde{I}_{ch,k} = I_{ch,k} \cup \left\{n^*\right\} \text{ for } \forall k \in I_{avail\_user} \tag{11}$$

$$k^* = \arg \max_{k \in I_{avail\_user}} \left[C_k\left(P_{\max}, \tilde{I}_{ch,k}\right) - C_k\left(P_{\max}, I_{ch,k}\right)\right] \tag{12}$$

$$I_{ch,k^*} = I_{ch,k^*} \cup \left\{n^*\right\} \tag{13}$$

Step 3: Delete the chunk in the set of available chunk i.e. $I_{avail\_chunk}=I_{avail\_chunk}-\{n^*\}$*.Repeat the steps 1, 2 and 3, until all chunks are allocated.*

Steps 1 and 2 assign each chunk to the user with the highest channel gain for that chunk. This method provides a near-optimum solution achieving multi-user diversity and frequency selective diversity. However, it is neither necessary nor sufficient for optimality.

B. Interleaved FDMA

Since each chunk in I-FDMA consists of distributed subcarriers, the channel quality between chunks becomes similar. Therefore, chunk allocations similar to the approach taken with L-FDMA do not provide significant improvements. Instead, we can obtain multi-user diversity by selecting users with higher channel gains over the entire bandwidth than others. To achieve this, we define a representative channel gain to noise ratio (CNR) for user k over the entire set of subcarriers:

$$\Omega_k \equiv \frac{1}{L} \sum_{l=1}^{L} \frac{H_{l,k}}{\sigma_l^2} \tag{14}$$

Then, we abstract only N best users as the candidates of chunk allocation.

$$I_{best} = \left\{a_1, a_2, \cdots, a_N\right\},\ a_i = \arg \max_k{}^{(i)}\, \Omega_k \tag{15}$$

where a_i is the user index of i-th best user with respect to Ω. Next, the objective is to find the number of assigned chunks for each user in set I_{best}. However, there is a restriction for chunk allocation as follows.

Property 1. The number of subcarriers assigned to each user should be a power of 2 in order to maintain equidistant subcarrier mapping and lower PAPR. It follows that the number of assigned chunks per user is also a power of 2 if the number of subcarriers per chunk is a power of 2. The details about restrictions of subcarrier allocation for I-FDMA are well explained in [8].

For I-FDMA, we propose a chunk allocation which determines the number of chunks for each user as follows.

Initialization: N best users with respect to Ω are added into the set of available users, i.e. $I_{avail\_user}=I_{best}$*, and the number of available chunks equals to N, i.e.* $N_{avail\_ch}=N$.

For each user from the first best user to the N th best user, the number of chunks is determined by steps 1 and 2 below. For user k^*,

Step 1. Assign the number of chunks for each user proportional to the ratio of the user's SNR to the sum of SNRs for all users. The assigned number is the highest power of two less than this ratio 2^x, $x \in \{1,2,\ldots,\log_2 N\}$.

$$N_{ch,k^*} = \left\lfloor N_{avail\_chunk} \cdot \frac{\Omega_{k^*}}{\sum_{j \in I_{avail\_user}} \Omega_j} \right\rfloor_{2^x} \tag{16}$$

For example, if there are two available users ($\Omega_1=3$, $\Omega_2=1$, $N_{avail\_chunk}=4$), user 1 has 3 (=4×3/4) chunks without flooring. Then, user 1 has 2 chunks after flooring.

Step 2.Delete the allocated user k from the set of available users and update the number of available chunks, i.e.* $I_{avail\_user} = I_{avail\_user} - \{k^*\}$, $N_{avail\_chunk}=N_{avail\_chunk}-N_{ch,k^*}$*. Repeat the steps 1 and 2, until all chunks are allocated.*

Chunks in I-FDMA have a tree structure as illustrated in Fig. 2 to maintain equidistant subcarrier mapping. As shown in Fig. 2, we can group chunks so that the subcarriers in the group are equidistant as in Fig. 1 (b) for example. If one of the sets in a level is allocated to a user, the "descendent" sets can't be assigned to other users due to the restriction of exclusive allocation.

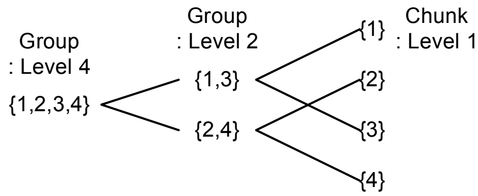

Fig. 2 An example of chunk tree structure (16 subcarriers, 4 chunks)

We choose the number of chunks assigned to each user obeying the restriction rule in property 1 via steps 1 and 2. Then, the following procedures referred from [8] are used for chunk/subcarrier mapping with equidistance.

Step 3. Chunk/Subcarrier mapping

- *Find the tree obeying the equidistance rule.*
- *Choose a user with the highest number of assigned chunks.*
- *Select a set of chunks in the level which equals the number of assigned chunks. Eliminate all "descendent" sets in the tree. Choose an available set for the user with the second highest number of assigned chunks and eliminate all "descendent" sets in the tree.*

Repeat for all assigned users.

IV. PERFORMANCE EVALUATIONS

Simulation results are provided employing channel-dependent scheduling derived in the previous section. We compare them with the performance of a round-robin scheduling scheme. We consider the multi-path fading channel in which a discrete time channel impulse response model is used as (17) where $T_s(=1/B)$ denotes the sampling time of multi-path component. Zeros are padded into the time domain so that the total length of $v(n)$ equals to L (=Number of subcarriers).

$$v(n) = \begin{cases} A \cdot \sqrt{P_{rel}(\tau_l)} \cdot w(\tau_l), & \text{if } nT_s = \tau_l \\ 0, & \text{Otherwise} \end{cases} \tag{17}$$

where $w(\tau_l)$ is a zero mean complex Gaussian noise process, and τ_l is the propagation delay of path l. A is a normalized parameter such that the average power, $E[\sum|v(n)|^2]$ equals to 1. $P_{rel}(\tau_l)$ is the relative power of path l. We consider a typical urban area propagation model with 6 tap setting specified in [12], and the parameters listed in table 1.

Table 1. Relative powers of delay profile [12]

Delay (μsec)	0.0	0.2	0.6	1.6	2.4	5.0
Rel.Pwr.(dB)	-3.0	0.0	-2.0	-6.0	-8.0	-10.0

The channel gain of subcarrier i for user k can be expressed as (18) where $L_{loss,k}$ denotes path-loss and shadowing of user k assumed to be a constant over all subcarriers. Thus, channel gain to noise ratio of subcarrier i can be represented as (20) where σ_i^2 is the noise power of subcarrier i in the receiver. By assuming $\sigma_i^2=\sigma^2$ for all subcarriers, path-loss/shadowing to noise ratio (PSNR) is defined as $G_k=1/(L_{loss,k}\sigma^2)$ which is constant over all subcarriers of user k. In our analysis, we assume that all users have the same PSNR.

$$H_{i,k} = \frac{|V_k(i)|^2}{L_{loss,k}}, \quad V_k(i) = \sum_{l=1}^{L} v_k(l) \exp\left[-\frac{2\pi i l}{T_s}\right] \tag{18,19}$$

$$\frac{H_{i,k}}{\sigma_i^2} \approx \frac{|V_k(i)|^2}{L_{loss,k}\sigma^2} \equiv G_k |V_k(i)|^2 \tag{20}$$

To evaluate the performance, we have simulated frequency selective fading of K users using (17) and obtain the average sum of user capacity derived from (6) after a large number of trials. In the figures, we use the following abbreviations: R-L-FDMA (N users are assigned to the N chunks by round-robin fashion), S-L-FDMA (The proposed CDS method for L-FDMA), R-I-FDMA (N users are assigned to the N chunks by round-robin fashion), and S-I-FDMA (The proposed CDS method for I-FDMA). Fig. 3 and Fig. 4 show the outage probability which is defined as P_r(upper bound on user data rate<target data rate). In this case, there are only N users in the system, while each of them has one chunk ($N=K$). Thus, all of users transmit their information in the same time interval. Since S-I-FDMA has the same performance as R-I-FDMA, we only show the results for R-I-FDMA. For the case of S-L-FDMA with N=8, some users may transmit with higher data rate by assigning a chunk with higher channel gain, whereas the scheduling may not be beneficial to other users by assigning a chunk in deep fading. Since I-FDMA provides interleaved subcarriers, users may avoid the worst allocation which assigns many adjacent subcarriers in deep fading. The variance of data rate between users decreases in case of I-FDMA due to spread spectrum diversity. This implies that the outage performance of I-FDMA can outperform S-L-FDMA. In Fig. 4, we can see that the frequency selective scheduling gain increases and L-FDMA outperforms I-FDMA at most PSNR cases, since the gain of frequency selective diversity overwhelms the gain of spread spectrum diversity in I-FDMA. Fig. 5 and 6 show the rate-sum capacity when all users share N chunks (K can be greater than N in these cases). We observe that the overall performance becomes better if there are more chunks since the transmit power per subcarrier is increased when a chunk is allocated to a user. Note that assigning small number of subcarriers to a user causes higher power per subcarrier and more bits per symbol. Segmenting more chunks is also advantageous in highly frequency selective environments due to higher resolution. As the number of users is increased, the gain of multi-user diversity is increased. Fig. 5 and 6 show that relative to round-robin, rate-sum capacity increases up to 130% for L-FDMA and 40% for I-FDMA by applying channel-dependent scheduling schemes. We also see that the rate sum capacity of R-L-FDMA is slightly higher than R-I-FDMA.

V. CONCLUSIONS

In this paper, we develop channel-dependent scheduling (CDS) schemes to increase the rate-sum capacity of uplink SC-FDMA and compare them with round-robin scheduling. For static subcarrier assignment (round robin scheduling), a system with users each transmitting at a moderate data rate is better off with I-FDMA. Due to the advantages of lower outage probability and lower PAPR, I-FDMA can be favorable for the subcarrier mapping technique of SC-FDMA. For CDS, L-FDMA has the potential for considerably higher data rate. Instead of increasing system capacity channel-dependent scheduling can be used with L-FDMA to reduce power consumption and PAPR. This is accomplished by using power control to establish a power margin. The effects of imperfect channel information and power margins on L-FDMA are challenging issues we are currently investigating.

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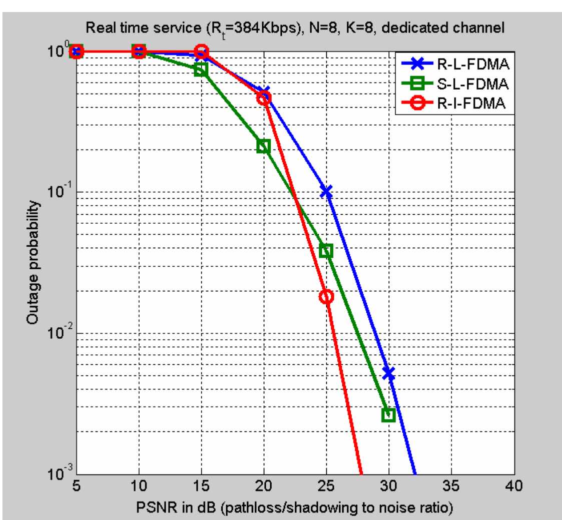

Fig. 3 Outage probability of real time service, target rate=384Kbps, # of simultaneous channel=8 (B=5MHz , L=256 subcarriers N = 8 chunks, K=8 users)

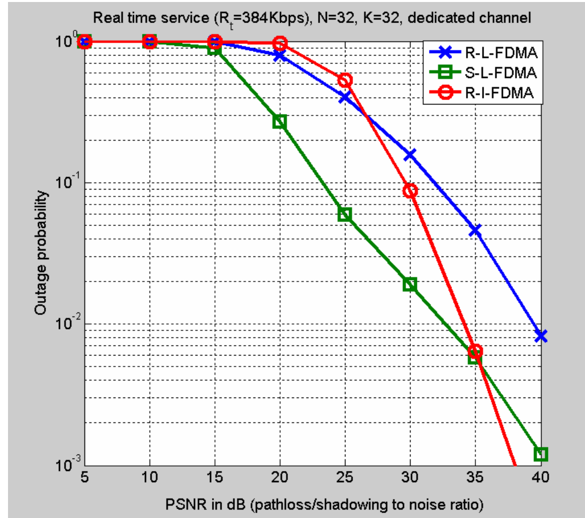

Fig. 4 Outage probability of real time users, target rate=384Kbps, # of simultaneous channel=32 (B=5MHz, L=256 subcarriers, N = 32 chunks, K=32 users)

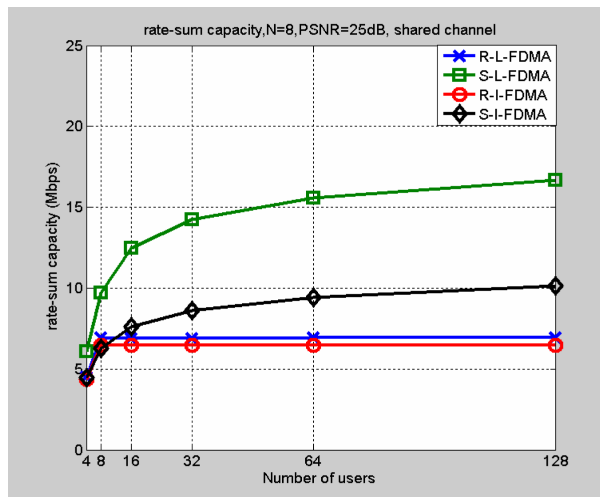

Fig. 5 Rate-sum capacity of shared users (B=5MHz, L=256 subcarriers, N=8 chunks, PSNR=25dB)

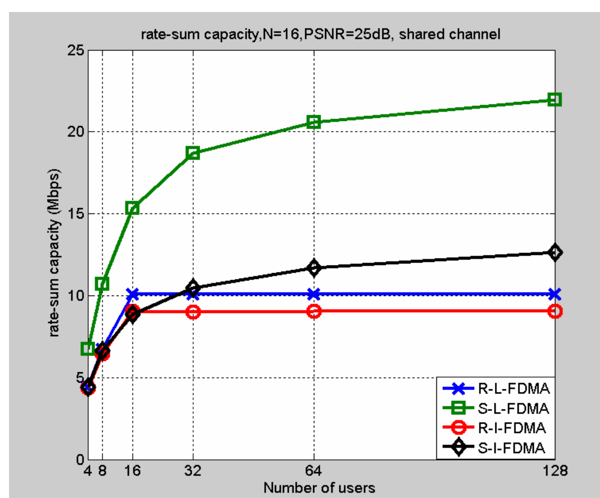

Fig. 6 Rate-sum capacity of shared users (B=5MHz, L=256 subcarriers, N=16 chunks, PSNR=25dB)